An improved calculation of the isospin-symmetry-breaking corrections to superallowed Fermi β decay

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In the determination of V_{ud}, an important strength of the nuclear measurements is that there are many $0^+ \rightarrow 0^+$ transitions available for study. It then becomes possible to validate the analysis procedure by checking that all transitions individually yield statistically consistent results for V_{ud}. Since the isospin-symmetry-breaking corrections depend on nuclear structure, they differ from transition to transition and are particularly sensitive to this consistency test. Thus the appearance of an anomalous result from any transition could signal a problem with the structure-dependent correction for that case, a problem which might have implications for other cases as well.

One disturbing development arose with the recent precise Penning-trap measurements [1, 2] of the Q_{EC} value for the superallowed decay of ^{46}V, which left the result for that transition more than two standard deviations away from the average of all other well-known transitions. This possible anomaly led us initially to reexamine the isospin-symmetry-breaking corrections for the ^{46}V transition, but what we learned from that reexamination prompted us to a more general reevaluation of the corrections for other transitions as well. The results of this reexamination have just been published [3].

Our previous shell-model calculations for ^{46}V considered six valence nucleons occupying the *pf*-shell orbitals outside a ^{40}Ca closed shell. However, an important part of the charge-dependent correction depends on the radial mismatch between the decaying proton in the parent nucleus and the resulting neutron in the daughter nucleus; but both these nucleons are bound to ^{45}Ti, so the structure of that nucleus turns out to be important too. What is most striking about ^{45}Ti is that it has a $3/2^+$ state at an excitation energy of only 330 keV, which is strongly populated in single-nucleon pick-up reactions like (p,d) and (^3He, α). Such low-lying *sd*-shell states can contribute to the structural parentage of the initial and final states of the superallowed transition and consequently must affect the radial mismatch between them. This indicated to us that a complete calculation of the isospin-symmetry-breaking correction for the decay of ^{46}V should include contributions from shells deeper than the *pf* shell.

The isospin-symmetry breaking correction, δ_C, is typically broken up into two pieces, $\delta_C = \delta_{C1} + \delta_{C2}$, of which the second, δ_{C2}, is the larger and more important component. It depends on the mismatch in the radial wave functions for the proton and neutron involved in the β transition. In ref. [3], this correction is evaluated from the formula

$$\delta_{C2} \approx \sum_{\pi^<,\alpha} S_\alpha^< \Omega_\alpha^< - \tfrac{1}{2} \sum_{\pi^>,\alpha} S_\alpha^> \Omega_\alpha^> \, , \qquad (1)$$

where S_α are spectroscopic factors for neutron pick-up in orbital α from, in our example, ^{46}Ti, while Ω_α is a measure of how much the radial overlap integral between the proton and neutron radial wave functions departs from unity. The sum over π is a sum over all "parent" states in ^{45}Ti that have significant parentage with the ground states of ^{46}Ti and ^{46}V. The superscripts < and > denote whether the isospin of the states in ^{45}Ti are isospin-lesser states, with $T_\pi = 1/2$, or isospin-greater states, with $T_\pi = 3/2$.

This equation provides the key to the strategy we used in calculating δ_{C2}. It demonstrates that there is a cancellation between the contributions of the isospin-lesser states and the isospin-greater states. Moreover, if the orbital α were completely full, then the Macfarlane and French sum rules [4] for spectroscopic factors would require $\sum_{\pi^<} S_\alpha^< = \frac{1}{2}\sum_{\pi^>} S_\alpha^>$ and the cancellation in Eq.(1) would be very strong. In fact, the cancellation would be complete if $\Omega_\alpha^< = \Omega_\alpha^>$. This cancellation is not in general complete because the radial-mismatch factors for isospin-lesser states are larger than those for isospin-greater states. Even so, cancellation is always significant, and it becomes most complete when closed-shell orbitals are involved. Thus, although the dominant contributions to δ_{C2} come from unfilled orbitals, we conclude that closed-shell orbitals must play a role, albeit one that decreases in importance as the orbitals become more deeply bound.

Based on these observations, our strategy was to use experiment to guide us in determining which closed-shell orbitals are important enough to include. Ideally, of course, one would take the spectroscopic factors determined from experiment and insert them into Eq.(1) but, especially where delicate cancellations are involved, the reliability of (forty-year-old) experimental spectroscopic factors is certainly not up to the task. Our strategy then was to use the shell model to calculate the spectroscopic factors but to limit the sum over orbitals α just to those for which large spectroscopic factors have been observed in neutron pick-up reactions.

Following along these lines, we have completed new calculations of δ_{C2} incorporating core orbitals in the shell model in cases where independent experimental information indicates that they are required. When combined with the other theoretical corrections and the experimental *ft* values we obtained a new set of corrected $\mathcal{F}t$ values. The agreement among the $\mathcal{F}t$ values for the thirteen well measured cases is very good and the consistency check imposed by the Conserved Vector Current hypothesis is well satisfied.

With these new corrections, the value of V_{ud} is increased by 0.04%, or by one standard deviation of the previous result [5]. With the new value, the sum of squares of the top-row elements of the CKM matrix is in perfect agreement with unitarity. However, this should not be regarded as the end of the story. Although there is excellent agreement with unitarity, it is the 0.1% uncertainty on the experimental sum that actually sets the critical limit on possible new physics beyond the standard model. This uncertainty can still be reduced by new precise experiments.

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